

## 1. Introduction

The theoretical description of combustion processes has experienced a steady and strong evolution over the past two decades. This development has been characterized by a crucial influence of thorough mathematics and, with some delay, computation. Both a parallel evolution of analytical tools and the significant occurrence of singular-perturbation regimes in combustion have been crucial for this development. The mutual influence and common evolution of physical theory and mathematical techniques initially spurned mostly the fundamental knowledge about the underlying mechanisms of combustion, while quantitative evaluation and engineering application were still out of reach. It was crucial to concentrate in the first place on problems simple enough to be understood in all detail, since only in this way was it possible to develop the mathematical tools needed. Yet, the problems considered suffered from oversimplification in comparison with real-life applications regarding both the detailed thermo-chemical models and the boundary and initial data considered.

More recently, and this is one of the motives for collecting this special issue of the *Journal of Engineering Mathematics*, the steady evolution of the theory of combustion has led to more directly applicable results which are now moving strongly into the realm of engineering:

- The advent of systematic reduction of chemical-kinetic schemes allowed one to transfer the mathematical tools of asymptotic analysis to realistic chemical kinetics, thereby replacing by and by the ‘old working horse’ of large activation-energy asymptotics. This is not to say, however, that this latter approach is outdated. Instead, it is still a most suitable *ansatz* when the focus is on new physical mechanisms that do not crucially depend on kinetics, but only need sufficient stiffness and state sensitivity to display their effects.
- Realistic equations of state and multi-phase models have been incorporated, making mathematical analyses available to a range of applications much wider than before.
- The numerical flow simulation of engineering problems has become a respectable scientific and engineering tool and, naturally, there is a desire to extend from pure flow simulation to the computation of combustion events. It is one of the emerging eminent characteristics of computation in combustion that analytical and computational tools are being combined in order to generate efficient algorithms. Due to the complexity of combustion physics, this ‘marriage’ is not only advantageous, but rather a necessary prerequisite.
- Important steps are made not only in the direction of engineering application, but also towards understanding the probably most challenging problem in combustion theory: The influence of turbulence. The advances in combustions theory made over the past decades are being combined with turbulence-modeling ideas. One particularly eminent role played by earlier detailed analyses of specialized flame-flow configurations is now to provide building blocks for the modeling of turbulent combustion.

The present special issue of the *Journal of Engineering Mathematics* provides a cross-section of these recent developments in the form of a collection of particular examples. Thus, Kim and Williams, using activation energy asymptotics, elaborate on the influence of non-equidiffusion on the extinction characteristics of diffusion flames. The contributions by Sanchez *et al.* and Niemann *et al.* deal with or use theories and techniques related to the systematic reduction of complex chemical-kinetic schemes. A theory regarding the formation of a radical pool in a high-temperature Hydrogen-Air mixing layer is presented by Sanchez *et al.* Obviously a suitable chemical-kinetic model, respecting the chain-branching character of hydrogen combustion in this regime, is called for. The authors succeed by incorporating a reduced four-step scheme that captures the chemical-kinetic essence. Niemann *et al.* discuss the development of a new efficient numerical technique aimed to overcoming the overwhelming storage requirements of numerically reduced kinetics in the framework of the ILDM (intrinsic lower-dimensional manifold) approach.

The contributions by Xu and Stewart, Margolis, Aldushin *et al.*, and Meinköhn and Sprengel share the problem of a suitable description of multiphase flow and combustion. Xu and Stewart compare various models for high-speed combustion and detonation of condensed phase explosives. Margolis analyses the combustion of porous energetic materials, implementing both a sophisticated two-phase flow model and a suitable chemical-kinetic description, allowing him to reveal the specific characteristics of the burning of nitramine-type propellants. Aldushin *et al.* work out the specific behavior of the buoyancy-driven filtration combustion, emphasizing the mutual feedback between the buoyancy-induced overall mass flux through the sample and the combustion temperature. Meinköhn and Sprengel consider the mechanism of heterogeneous combustion. A convection-diffusion-dominated film of reaction products establishes whose behavior controls the combustion process. They derive a long-wave approximation for the film dynamics, using tools of modern asymptotic analysis.

The contributions by Minaev and Sivashinsky, and Buckmaster deal with the intrinsically multi-dimensional behavior of premixed and non-premixed flames, respectively. Minaev and Sivashinsky study the stability of a premixed wedge-shaped flame embracing a wedge of unburnt gas. According to earlier arguments by Zel'dovich, there should be a corresponding wedge-shaped stagnation zone trailing the flame wedge. Minaev and Sivashinsky show that the Kelvin-Helmholtz instability of the slip surface bounding the stagnation wedge tends to aggravate the Darrieus-Landau instability of the adjacent branches of premixed flame. Buckmaster considers 'edge flames' that may establish at the termination point of a non-premixed flame. A prominent example are the so-called triple flames whose importance for theories of turbulent non-premixed combustion is discussed.

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